Day #11: Aug 7, 2019
Little piece on non-significance

p. 145 Table 3.3
P-value “moderate”

*FEV(iii):* A moderate *p* value is evidence of the absence of a discrepancy *γ* from *H*₀, only if there is a high probability the test would have given a worse fit with *H*₀ (i.e., smaller *P*-value) were a discrepancy *γ* to exist.

For a Fisherian like Cox, a test’s power only has relevance pre-data, they can measure “sensitivity”.

In the Neyman-Pearson theory of tests, the sensitivity of a test is assessed by the notion of *power*, defined as the probability of reaching a preset level of significance ...for various alternative hypotheses. In the approach adopted here the assessment is via the distribution of the random variable *P*, again considered for various alternatives (Cox 2006, p. 25)
$H_0: \mu \leq 150$ vs. $H_1: \mu > 150$ (Let $\sigma = 10$, $n = 100$)

M has to be 152 to reach the .025 level of significance, so M = 151 would miss it

Computation for SEV(T, M = 151, C: $\mu \leq 150$)
$Z = (151 - 150)/1 = 1$

$Pr(Z > 1) = .16$

SEV(C: $\mu \leq 150$) = low (.16).

• So there’s poor indication of $H_0$
Can they say $M = 151$ is a good indication that $\mu \leq 150.5$?

No, $\text{SEV}(T, M = 151, C: \mu \leq 150.5) = \sim .3$.

$\Pr(M > 151; \mu = 150.5)$

$\Pr(Z > 151 - 150.5) = \Pr(Z > .5) = .3$

But $M = 151$ is a good indication that $\mu \leq 152$

$[Z = 151 - 152 = -1; \ Pr(Z > -1) = .84 ]$

$\text{SEV}(\mu \leq 152) = .84$

It’s an even better indication $\mu \leq 153$ (Table 3.3, p. 145)

$[Z = 151 - 153 = -2; \ Pr(Z > -2) = .97 ]$
\[ \Pi(\gamma) : \text{"sensitivity function"} \]

Computing \[ \Pi(\gamma) \] views the P-value as a statistic.

\[ \Pi(\gamma) = \Pr(P \leq p_{\text{obs}}; \mu_0 + \gamma). \]

The alternative \( \mu_1 = \mu_0 + \gamma \).

Given that P-value inverts the distance, it is less confusing to write \( \Pi(\gamma) \)

\[ \Pi(\gamma) = \Pr(d > d_0; \mu_0 + \gamma). \]

Compare to the power of a test:

\[ \text{POW}(\gamma) = \Pr(d > c_\alpha; \mu_0 + \gamma) \text{ the N-P cut-off } c_\alpha. \]
FEV(ii) in terms of $\Pi(\gamma)$

*P-value is modest (not small):* Since the data accord with the null hypothesis, FEV directs us to examine the probability of observing a result more discordant from $H_0$ if $\mu = \mu_0 + \gamma$:

If $\Pi(\gamma) = \Pr(d > d_0; \mu_0 + \gamma)$ is very high, the data indicate that $\mu < \mu_0 + \gamma$.
Here $\Pi(\gamma)$ gives the severity with which the test has probed the discrepancy $\gamma$. 

**FEV/SEV** A small $P$-value indicates discrepancy $\gamma$ from $H_0$, if and only if, there is a high probability the test would have resulted in a larger $P$-value were a discrepancy as large as $\gamma$ absent.

**FEV/SEV** A moderate $P$-value indicates the absence of a discrepancy $\gamma$ from $H_0$, only if there is a high probability the test would have given a worse fit with $H_0$ (i.e., a smaller $P$-value) were a discrepancy $\gamma$ present.